1st Lecture

THEORY OF FUNDAMENTAL INTERACTIONS

Topic: the fundamental structure of matter, i.e. elementary particles & their interactions

Method: applied quantum field theory
[heuristic explanations, some proofs, not mathem. rigorous]

Disclaimer: no QFT course from the ground up
little about particle phenomenology
leaves out gravity $\leftrightarrow$ general relativity
not very rigorous or deductive
not a manual for future expert

Idea: explain the key concepts of the "Standard Model of Particle Physics"

Literature: follow just one book

"Digestible Quantum Field Theory"

by Andrei Smilga, Springer 2017

- is semi-popular, structured like a dinner
- treats subject 3 times, with increasing level of depth & complexity, plus toolbox part:

- Level 1: The Universe as we know it (1 lecture)
- Level 2: The edifice of physical theories, Bird's eye view of the Standard Model (4 lectures)
- Toolbox: Groups & algebras, Lagrangians & Hamiltonians, cross sections & amplitudes (3 lectures)
- Level 3: (6 lectures)
  - Fermion fields
  - Feynman graphs
  - Quantum chromodynamics
  - Electroweak interactions

prerequisties:

- math: elementary analysis & linear algebra (1st year university)
- physics: classical (analytical) mechanics, E&M, quant. mech.

contact me:

olaf.lechtenfeld@itp.uni-hannover.de

UNITS

SI: $\vec{F} = \dfrac{qQ}{4\pi\varepsilon_0}\dfrac{\vec{r}}{r^3}$ $\qquad [Q] = C = As$

CGS: $\vec{F} = qQ\,\vec{r}/r^3$ $\qquad [Q] = esu = g^{1/2}\,cm^{3/2}\,s^{-1}$

natural: $\hbar = c = 1$ $\qquad [Q] = 1$

Heaviside units
$Q \to Q\cdot\sqrt{4\pi}$
$\vec{F} = \dfrac{qQ}{4\pi}\,\vec{r}/r^3$
$\vec{\nabla}\cdot\vec{E} = \rho$
$\alpha = e^2/\hbar c \to e^2/4\pi$

Compton wavelength $\ell_c = \dfrac{h}{mc} \overset{\hbar=c=1}{=} \dfrac{2\pi}{m}$

$[L] = [T] = [M]^{-1} = [E]^{-1} = [p]^{-1}$

convenient unit: $1\ \text{eV} \approx 1.6\times 10^{-19}\ J$

$\rightsquigarrow m_e \approx 511\ \text{keV}, \quad m_p \approx 938\ \text{MeV}, \quad m_Z \approx 91.19\ \text{GeV}$

$\rightsquigarrow 1\ \text{fm} = 10^{-15}\ m \approx 3.3\times 10^{-24}\ s \approx (200\ \text{MeV})^{-1}$

Planck units: $\hbar = c = m_{pl} = 1$ $\rightsquigarrow$ no dim's left

$G_N = \dfrac{\hbar c}{m_{pl}^2} \rightsquigarrow m_{pl} \approx 1.22\times 10^{19}\ \text{GeV} \approx 2.2\times 10^{-5}\ g$

$\rightsquigarrow m_e \approx 4.2\times 10^{-23}\ m_{pl}$

$\ell_{pl} = \sqrt{\dfrac{\hbar G_N}{c^3}} \approx 1.6\times 10^{-35}\ m, \quad t_{pl} = \ell_{pl}/c \approx 5.4\times 10^{-44}\ s$

THE UNIVERSE AS WE KNOW IT

current understanding in length scales ranges from $\sim 10^{-18}$ m to 10^{26} m

10^{-8} $\varnothing$ atom — observable universe

beginning $\sim$ 13.77 Gyr ago, expanding

understand dynamics after $t \approx 10^{-10}$ s

$T \approx 100\,\text{GeV} \approx 10^{15}$ K

evolution well described by classical GR (Friedmann eq.)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi}{m_{pl}^2} T_{\mu\nu} \overset{SI}{=} \frac{8\pi G_N}{c^4} T_{\mu\nu}$$

need to know energy-momentum ($\& \Lambda$) $\Longrightarrow$ matter!

Standard Model of Particle Physics

Gravitational Interaction

- is 43 orders of magn. weaker than electr. repulsion

  $m_e^2/\alpha \approx (4.2\times10^{-23}\, m_{pl})^2/(137)^{-1} \approx 2.4\times10^{-43}$
- universal & attractive $\rightarrow$ any form of energy
- nonrel. limit (Kepler, Newton):

  $\vec{F} = -G_N\, m M\, \vec{r}/r^3$
- grav. field influences time measurements:

  clocks tick slower in a grav. field

  $\Delta t \approx (45-7)\,\mu s/\text{day} \approx 38\,\mu s/\text{day}$ for GPS satellite
- most spectacular: neutron stars, black holes & vicinity
- Newtonian calculation for radius of a mass M such that the escape velocity equals c yields

  $r_g = 2G_N M/c^2$ [earth: 1 cm, sun: 3 km, SgA: 3×10^6 km]

  this grav. radius agrees exactly with Schwarzschild radius
- black-hole density $\rho \sim 1/M^2$ [sun: $10^{16}\, g/cm^3$, quasars: $10^{-2}\, g/cm^3 <$ water]

Electromagnetic Interactions

- both attractive & repulsive ($qQ \lessgtr 0$)
- long-ranged, but (mostly) irrelevant on macroscales
- +ve & −ve charges almost always balanced out
  → only higher multipoles, no role on cosmic scale
- but crucial for microscopic structure of matter
- characteristic radius of atom $l_{at} \sim 1/m_e\alpha \sim 1\,\text{Å}$
- all everyday forces (like pressure, support, friction) from interactions of large sets of electrons in contacting bodies
- first-principle estimate of matter (say hydrogen) density:
  $$\rho_H \sim m_p / (3\cdot 2 l_{at})^3 \sim m_p (m_e\alpha)^3/200 \approx 0.05\ g/cm^3$$
  half the actual value
- matter can be squeezed:
  $\rho_{\text{sun center}} \approx 150\ g/cm^3$, $\rho_{\text{neutron star}} \approx 8\times 10^{14}\ g/cm^3$
- breaking strength ~ atom. shell energy/vol ~ $m_e \alpha^2 (m_e\alpha)^3 \approx 3\times 10^{13}\ Pa$ (10 × diamond)
- not least: radiation (light, radio, microwaves etc.) is electromagnetic

Strong Interactions

- responsible for structure of nuclei
  made of nucleons = protons or neutron
- characteristic scale $\ell_s \sim 1$ fm $\sim$ range of strong force
  falls off exponentially fast at larger distances
- fundamental constant has dimension of mass (energy),
  we can take $m_p \approx 936$ MeV or $(1\,\text{fm})^{-1} \approx 200$ MeV
- first-principle estimate of stellar masses:
  max. possible mass before stellar black-hole collapse when
  grav. binding energy $\sim$ mass energy $\Leftrightarrow \frac{G_N M^2}{R} \sim M$
  moreover, $M = \rho R^3$ and $\rho_{\text{nuclear}} \sim m_p/\ell_s^3 \sim m_p^4$, $G_N = \frac{1}{m_{p\ell}^2}$
  $\Rightarrow M \sim m_p^4 R^3$ and $M \sim R/G_N \sim m_{p\ell}^2 R$
  $\Rightarrow \frac{M^3}{M} \sim \frac{m_{p\ell}^6 R^3}{m_p^4 R^3} \Rightarrow M \sim m_{p\ell}^3/m_p^2 \approx 3.7 \times 10^{30}$ kg
  ($M_\odot \approx 2 \times 10^{30}$ kg)
- astrophysics: $M_{\text{crit}} \approx 3 M_\odot$ = Landau-Oppenheimer-Volkoff limit
  stability of white dwarfs: $\bar{M} \approx 1.4 M_\odot$ = Chandrasekhar limit

- nuclear structure:

pn, ppn, ppnn stable ; pp, pnn, pppn unstable

pn attraction stronger than pp attraction (Pauli principle)

$\exists$ Coulomb repulsion

- estimate stability of nuclei

$$Z = \#p \,, \quad A = \#p + \#n$$

strong binding energy $\sim -A$ (sees neighbors only)

electrom. repulsion energy $\sim \alpha Z^2$ (long-range pair interaction)

rough stability condition: $\alpha Z^2 - A < 0$

$\rightsquigarrow$ large nuclei ($A \gtrsim 20$) need excess of neutrons

- what about neutronic matter?

stability of nn, nnn, n^{billion} $\longrightarrow$ neutron stars?

needs weak interactions...

Weak Interactions

- β-decay = emission of electrons from the nucleus

  are e^- hidden inside nucleus? no: Heisenberg uncertainty!

  $\rightsquigarrow$ e^- localized in $(1\,\mathrm{fm})^3$ is ultrarelativistic $\rightsquigarrow$ escape
- electrons are <u>created</u> during β-decay:

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

$\bar{\nu}_e \longrightarrow$ anti-neutrino, interacts very weakly discovered in 1956

this process destabilizes nuclei with too many neutrons

- stability of neutrons in nuclei: binding energy!

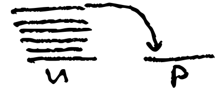
  $m_n - m_p - m_e \approx 1.25\ \mathrm{MeV}$ for <u>free</u> neutron, but

$$E^{\text{binding}}_{^4He} - E^{\text{binding}}_{^3He} \approx 28\ \mathrm{MeV} - 8\ \mathrm{MeV} \gg 1.25\ \mathrm{MeV}$$

so $^4He \longrightarrow {}^3He + p + e^- + \bar{\nu}_e$ is forbidden by energy conserv.

but with rich enough neutron content, β-decay possible

- so why are neutron stars stable?

  Pauli principle:  + gravity keeps electrons inside ($\leftarrow$ thermodynamics)

  $\longrightarrow p + e^- \longrightarrow n + \nu_e$ favored

- weak interactions instrumental for getting rid of 2nd & 3rd generation particles, e.g.

$$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

similar for τ & s, c, b, t quarks
also $W^\pm$, Z, H decay weakly but into two particles

- only weak <u>decays</u> ?

no: rearrange the "chemical process" to get interactions:

$$\nu_\mu + e^- \longrightarrow \nu_e + \mu^- \quad \text{or} \quad \bar{\nu}_e + p \longrightarrow n + e^+$$

- why "weak" interactions?

probabilities are much smaller than for electrom. interactions
weak decays are also less probable $\longrightarrow$ longer lifetimes

$\tau_\mu \approx 2.2\ \mu s \gg \tau_{em} \approx 10^{-17} s \gg \tau_{strong} \approx 10^{-23} s$
$\tau_n \approx 17$ min

- for life on our cozy planet all four interactions are crucial.
shifting their properties just a little bit appears to be deadly